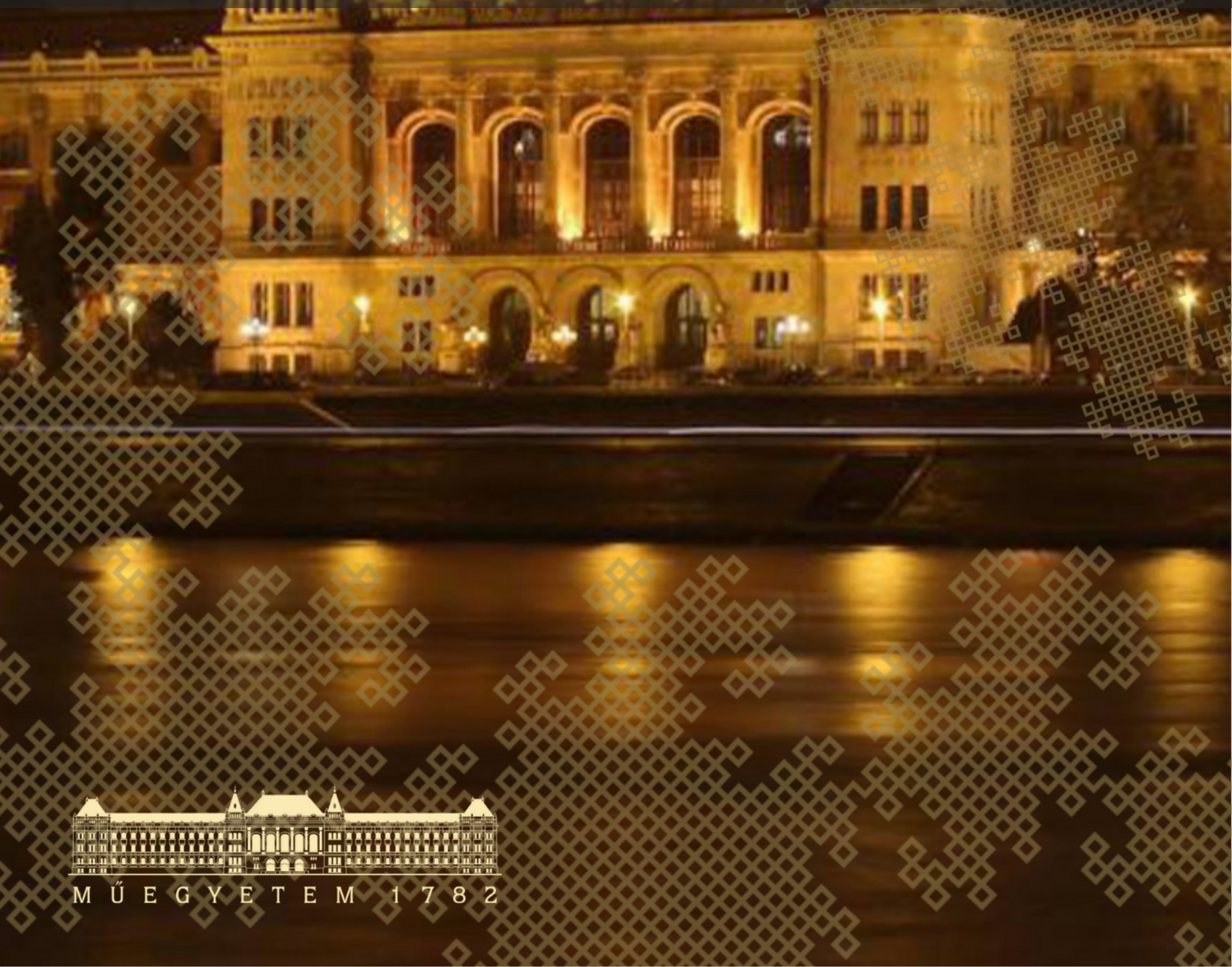
Geometry of Deterministic and Random Fractals

Honouring the 60+1st birthday of
Professor Károly Simon

27th June 2022 - 1st July 2022

Budapest University of Technology and Economics

Book of abstracts



Geometry of Deterministic and Random Fractals

Honouring the 60+1st birthday of Professor Károly Simon

Book of abstracts

Organisers

Balázs Bárány
Budapest University of Technology and Economics

István Kolossváry
University of St. Andrews

Roland Molontay
Budapest University of Technology and Economics

Marcell Nagy
Budapest University of Technology and Economics

R. Dániel Prokaj
Budapest University of Technology and Economics

Michał Rams
Polish Academy of Sciences

Andrea Széki
Budapest University of Technology and Economics

M Ű E G Y E T E M 1 7 8 2

Preface

The workshop focuses on the recent developments of geometric measure theory, dimension theory of dynamical systems, the geometry of random and deterministic fractals, and related topics. It also provides an ideal occasion to discuss the new directions of the research, to exchange ideas, and to evolve new collaborations. The workshop is dedicated to celebrating the 60+1st birthday of Károly Simon.

Budapest University of Technology and Economics
Budapest, June 2022

Monday	Tuesday	Wednesday	Thursday	Friday
8:30-9:00 Registration				
9:00-9:15 Opening	**9:00-9:45** F. Przytycki	**9:00-9:45** M. Hochman	**9:00-9:45** T. Jordan	**9:00-9:45** P. Raith
9:15-10:00 B. Solomyak				
	9:55-10:25 N. Jurga	**9:55-10:25** A. Rapaport	**9:55-10:25** Z. Buczolich	**9:55-10:25** S. Baker
10:10-10:40 T. Kucherenko				
	10:30-11:00 *break*	**10:30-11:00** *break*	**10:30-11:00** *break*	**10:30-11:00** *break*
10:45-11:15 *break*				
	11:00-11:45 M. Urbański	**11:00-11:45** P. Shmerkin	**11:00-11:45** K. Falconer	**11:00-11:30** D. Allen
11:15-12:00 P. Mattila				
				11:40-12:25 M. Pollicott
	11:55-12:25 C. Wolf	**11:55-12:10** A. Yavicoli	**11:55-12:25** F. Ledrappier	
12:10-12:25 W. O'Regan		**12:10-14:00** *lunch*		
12:30-14:15 *lunch*	**12:30-14:15** *lunch*		**12:30-14:15** *lunch*	**12:35– Farewell**
		14:00– Excursion to Székesfehérvár		
14:15-15:00 J. Komjáthy	**14:15-14:45** A. Käenmäki		**14:15-14:45** P. Allaart	
	14:55-15:10 A. Śpiewak		**14:55-15:10** A. Banaji	
15:15-15:30 K. Czudek	**15:15-15:30** A. Soós*		**15:15-15:30** A. Rutar	
15:35-16:05 *break*	**15:35-16:05** *break*		**15:35-16:05** *break*	
16:05-16:20 R. Miculescu	**16:05-16:20** X. Zhang		**16:05-16:20** Á. Farkas	
16:25-16:40 V. Siwach	**16:25-16:40** J. Ratsaby		**16:25-16:40** T. Jones	
16:45-17:00 M. Tripathi	**16:45-17:00** C. Wormell		**16:45-17:00** B. Ward	
17:05-17:20 S. Hayes*	**17:05-17:20** S. Kittle		**17:05– Reception**	
	17:25-17:40 V. Agrawal*			

1

Schedule

Monday

8:30-9:00
Registration

9:00-9:15
Opening

9:15-10:00
Boris Solomyak: *(Almost) thirty years of transversality method*

10:10-10:40
Tamara Kucherenko: *Flexibility of the Pressure Function*

10:45-11:15
Break

11:15-12:00
Pertti Mattila: *Hausdorff dimension of level sets and intersections*

12:10-12:25
William O'Regan: *Efficiently covering carpets with tubes*

12:30-14:15
Lunch

14:15-15:00
Júlia Komjáthy: *One-neighborhood biased first passage percolation on scale-free spatial random graphs*

15:15-15:30
Klaudiusz Czudek: *Counterexamples to the central limit theorem for random rotations*

15:35-16:05
Break

16:05-16:20
Radu Miculescu: *The structure of fuzzy fractals generated by an orbital fuzzy iterated function system*

16:25-16:40
Vijay Siwach: *Rational Spline Zipper α-Fractal Functions*

16:45-17:00
Mohit Tripathi: *Fractal Interpolation for Data Set with Stable Noise*

17:05-17:20
Sandra Hayes: *TBA*

Tuesday

9:00-9:45
Feliks Przytycki: *No hyperbolic subsets in solenoidal sets for quadratic polynomials*

9:55-10:25
Natalia Jurga: *Parabolic carpets*

10:30-11:00
Break

11:00-11:45
Mariusz Urbański: *The exact value of Hausdorff dimension of escaping sets of class B of meromorphic functions*

11:55-12:25
Christian Wolf: *Computability in Dynamical Systems*

12:30-14:15
Lunch

14:15-14:45
Antti Käenmäki: *Finer geometry of planar self-affine sets*

14:55-15:10
Adam Śpiewak: *Typical absolute continuity for classes of dynamically defined measures*

15:15-15:30
Anna Soós: *TBA*

15:35-16:05
Break

16:05-16:20
Xintian Zhang: *Hausdorff dimension of eventually always hitting set of self similar IFS with SSC*

16:25-16:40
Joel Ratsaby: *Bounded-complexity approximation of filled-Julia sets*

16:45-17:00
Caroline Wormell: *Linear response for high-dimensional systems and mixing of Cantor sets*

17:05-17:20
Samuel Kittle: *Absolute Continuity of Self Similar Measures*

17:25-17:40
Vishal Agrawal: *TBA*

Wednesday

9:00-9:45
Michael Hochman: *An entropy version of Bourgain's projection theorem*

9:55-10:25
Ariel Rapaport: *Dimension of self-affine measures in* $\mathbb{R}^d$

10:30-11:00
Break

11:00-11:45
Pablo Shmerkin: *Slices and tubes through self-similar sets*

11:55-12:10
Alexia Yavicoli: *Thickness and a Gap Lemma in* $\mathbb{R}^d$

12:10-14:00
Lunch

14:00–
Excursion to Székesfehérvár

Thursday

9:00-9:45
Thomas Jordan: *Lyapunov dimension away from the strongly irreducible case*

9:55-10:25
Zoltán Buczolich: *Measures, annuli and dimensions*

10:30-11:00
Break

11:00-11:45
Kenneth Falconer: *Projections or Random Images of Fractals - What's the Difference?*

11:55-12:25
Francois Ledrappier: *Exact dimension of the Furstenberg measure*

12:30-14:15
Lunch

14:15-14:45
Pieter Allaart: *Density spectrum of Cantor measure*

14:55-15:10
Amlan Banaji: *Dimensions of infinitely generated self-conformal sets*

15:15-15:30
Alex Rutar: *Classifying Dimension Spectra*

15:35-16:05
Break

16:05-16:20
Ábel Farkas: *A hike through the ridge of the open set condition*

16:25-16:40
Taylor Jones: *On the Box Counting Dimension of a Class of Random Self-Affine Functions*

16:45-17:00
Benjamin Ward: *On the measure of Badly approximable sets*

17:05–
Reception

Friday

9:00-9:45
Peter Raith: *On fractal dimensions of invariant sets for low dimensional dynamical systems*

9:55-10:25
Simon Baker: *Overlapping iterated function systems from the perspective of Metric Number Theory*

10:30-11:00
Break

11:00-11:30
Demi Allen: *Weighted approximation in higher dimensional missing digit sets*

11:40-12:25
Mark Pollicott: *Estimating dimension, exponents and drift*

12:35–
Farewell

2 Abstracts

Vishal Agrawal (Indian Institute of Technology (BHU), Varanasi, India)
TBA
Abstract: TBA

Pieter Allaart (University of North Texas, USA)
Density spectrum of Cantor measure
Abstract: Given a number $\rho \in (0, 1/2)$, the "middle $1 - 2\rho$" Cantor set C_ρ in $[0, 1]$ has Hausdorff dimension $s = -\log 2/\log \rho$, and the uniform distribution μ_ρ on C_ρ is just s-dimensional Hausdorff measure restricted to C_ρ. One might ask what the possible values are of the lower and upper s-densities

$$\liminf_{r\to 0} \frac{\mu_\rho(B(x, r))}{(2r)^s}, \qquad \limsup_{r\to 0} \frac{\mu_\rho(B(x, r))}{(2r)^s}.$$

Feng, Hua and Weng (2000) showed that for $\rho \leq 1/3$ these upper and lower densities can be expressed in terms of the function

$$\tau(x) = \min\{\liminf_{n\to\infty} T^n(x), \liminf_{n\to\infty} T^n(1 - x)\},$$

where $T\colon [0, \rho]\cup[1-\rho, 1] \to [0, 1]$ is the 2-to-1 piecewise linear map whose inverse branches are $x \mapsto \rho x$ and $x \mapsto \rho x + 1 - \rho$. Thus it is of interest to study the set $\Gamma := \{\tau(x) : x \in C_\rho\}$. In this talk we show that Γ is a compact set with the same dimension as C_ρ which is closely related to both the set of kneading invariants of unimodal maps and the set of univoque numbers in non-integer base expansions. We prove the identity

$$\dim_H(\Gamma \cap [t, 1]) = \dim_H\{x \in C_\rho : \tau(x) \geq t\},$$

and calculate the Hausdorff dimension of the level sets of $\tau(x)$. The results are directly applicable to the densities of Bernoulli convolutions with parameter $\rho \in (0, 1/3]$. (Joint work with D. Kong.)

Demi Allen (University of Exeter, UK)
Weighted approximation in higher dimensional missing digit sets
Abstract: In this talk I will discuss joint work with Ben Ward (York, UK) in which we obtain bounds for the Hausdorff dimension of sets of weighted simultaneously well-approximable points contained within certain higher dimensional "missing digit sets" (i.e. Cantor-type sets in $\mathbb{R}^n$). Our results are achieved by applying the recent mass transference principle for rectangles proved by Wang and Wu [Mathematische Annalen, 2021] to results on the Hausdorff measure of sets of (non-weighted) well approximable points in self-conformal and self-similar sets which can be deduced from work by myself and Balázs Bárány (Budapest, Hungary) [Mathematika, 2021].

Amlan Banaji (University of St Andrews, UK)
Dimensions of infinitely generated self-conformal sets
Abstract: Dimension interpolation involves taking two notions of dimension and finding a continuously parameterised family of dimensions which lie between them. Infinitely generated self-conformal sets,

such as sets of irrational numbers whose continued fraction expansions have restricted entries, are a natural case for investigation. For these fractal sets, we will give a precise description of the possible behaviours of the intermediate dimensions (which lie between Hausdorff and box dimension) and Assouad spectrum (which lies between box and Assouad dimension). This is based on joint work with Jonathan Fraser.

Simon Baker (University of Birmingham, UK)
Overlapping iterated function systems from the perspective of Metric Number Theory
Abstract: Khintchine's theorem is a classical result from metric number theory which relates the Lebesgue measure of certain limsup sets with the divergence of naturally occurring volume sums. Importantly this result provides a quantitative description of how the rationals are distributed within the reals. In this talk I will discuss some recent work where I prove that a similar Khintchine like phenomenon occurs typically within many families of overlapping iterated function systems. Families of iterated function systems these results apply to include those arising from Bernoulli convolutions, the 0,1,3 problem, and affine contractions with varying translation parameters.

Time permitting I also will discuss a particular family of iterated function systems for which we can be more precise. Our analysis of this family shows that by studying the metric properties of limsup sets, we can distinguish between the overlapping behaviour of iterated function systems in a way that is not available to us by simply studying properties of self-similar measures.

Zoltán Buczolich (ELTE Eötvös Loránd University, HU)
Measures, annuli and dimensions
Abstract: Given a Radon probability measure μ supported in $\mathbb{R}^d$, we are interested in those points x around which the measure is concentrated infinitely many times on thin annuli centered at x. Depending on the lower and upper dimension of μ, the metric used in the space and the thinness of the annuli, we obtain results and examples when such points are of μ-measure 0 or of μ-measure 1. This is a joint work with Stéphane Seuret. https://arxiv.org/abs/2111.09379

Klaudiusz Czudek (Nicolaus Copernicus University, PL)
Counterexamples to the central limit theorem for random rotations.
Abstract: Fix some irrational angle of rotation α, and consider a random walk in which at every step one jumps either to $x + \alpha$ or to $x - \alpha$ with equal probabilities, provided the current position is x. The talk will be devoted to the cental limit theorem for additive functionals of this Markov chain. If the angle is Diophantine and observable is smooth, then the central limit theorem holds. Given Liouville angle, I shall sketch a construction of smooth observable with failure of the central limit theorem. For some angles it is possible to construct an analytic observable.

Kenneth Falconer (University of St. Andrews, UK)
Projections or Random Images of Fractals - What's the Difference?
Abstract: The talk will compare and contrast projections and random images of fractals, in particular in relation to dimension properties. The talk will draw on results, both recent and less recent, by the speaker and others.

Ábel Farkas (Rényi Institute, HU)
A hike through the ridge of the open set condition
Abstract: I will show interesting examples of self-similar sets that enlighten differences between the OSC and the non-OSC case.

Sandra Hayes (The City University of New York Graduate Center, USA)
TBA
Abstract: TBA

Michael Hochman (Hebrew Univ. Jerusalem, IS)
An entropy version of Bourgain's projection theorem
Abstract: I will discuss joint work with Elon Lindenstrauss and Peter Varju in which we prove entropy versions of Bourgain's projection theorem (related to the sum-product theorem).

Taylor Jones (University of North Texas, USA)
On the Box Counting Dimension of a Class of Random Self-Affine Functions
Abstract: In 2005, H. Okamoto defined a family of continuous functions which have interesting differentiability properties based on a parameter a. In this talk we introduce a random process to the construction of a more general class related to Okamoto functions. We answer some natural questions about this class of functions, such as continuity and differentiability conditions. In particular we will use martingale theory to give an explicit formula for the almost sure box-counting dimension of their graphs.

Thomas Jordan (University of Bristol, UK)
Lyapunov dimension away from the strongly irreducible case
Abstract: The Lyapunov dimension of a measure is a quantity which always gives the upper bound of the Hausdorff dimension of an ergodic shift invariant measure projected onto a self-affine iterated function system. We will show how this upper bound can be shown (joint work with Mark Pollicott and Károly Simon). We then look at a simple example where it can be exactly determined when the Lyapunov dimension is the Hausdorff dimension. We will then look at how work on the Hausdorff dimension of ergodic measures on self-similar sets (joint with Ariel Rapaport) can be used to look at dimensions of ergodic measures on diagonal self-affine sets (using work of Feng and Hu) and also on diagonal/anti-diagonal systems (joint work with Jonathan Fraser and Natalia Jurga).

Natalia Jurga (University of St Andrews, UK)
Parabolic carpets
Abstract: We introduce a family of non-conformal and non-uniformly contracting iterated function systems. We refer to the attractors of such systems as parabolic carpets. Roughly speaking they may be thought of as nonlinear analogues of self-affine carpets which are allowed to have parabolic fixed points. In this talk we will consider their box dimensions as well as discussing directions for further investigation.

Antti Käenmäki (University of Oulu, FI)
Finer geometry of planar self-affine sets
Abstract: For a planar self-affine set satisfying the strong separation condition, it has been recently proved that under mild assumptions the Hausdorff dimension equals the affinity dimension. In this article, we continue this line of research, and our objective is to acquire more refined geometric information in this setting. In a large class of such non-carpet planar self-affine sets, we characterize Ahlfors regularity, determine the Assouad dimension of the set and its projections, and estimate the Hausdorff dimension of slices. We also demonstrate that the Assouad dimension is not necessarily bounded above by the affinity dimension. The talk is based on a joint work with Balázs Bárány and Han Yu.

Samuel Kittle (University of Cambridge, UK)
Absolute Continuity of Self Similar Measures
Abstract: We prove that a self similar measure is absolutely continuous providing that it satisfies a condition depending on its Garsia entropy, contraction ratio, and the separation between different points in approximations of the self similar measure. In the special case of Bernoulli convolutions this paper gives a sufficient condition for the Bernoulli convolution with parameter lambda to be absolutely continuous in terms of lambda and its Mahler measure.

Júlia Komjáthy (Delft University of Technology, NL)
One-neighborhood biased first passage percolation on scale-free spatial random graphs
Abstract: In this talk we consider first-passage percolation in which the transmission time between two nodes is non-iid, but vertex-dependent. Namely, the transmission time increases by a penalty factor polynomial in the expected degree of the vertices, to model limited time and awareness of nodes with large degree. We consider two spatially embedded l scale-free random graph models: (finite and infinite) Geometric Inhomogeneous Random Graphs, and Scale-Free Percolation. In these spatial models embedded in $\mathbb{R}^d$, the connection probability between two vertices depends on their distance and on their expected degrees. We find that the size of the cluster of nodes to which the transmission is successful before time t, $I(t)$, as a function of t, undergoes three phase transitions as the penalty exponent of the transmission times changes, and deviates more and more from classical first passage percolation. For

small penalties, we give the criterion for the model to be explosive, i.e., in finite time the transmission reaches infinitely many nodes. As the penalty exponent gradually increases, $I(t)$ becomes finite almost surely, and the phases are: $I(t)$ is either stretched-exponential in t, or polynomial in t, but strictly faster than what the dimension allows; or polynomial proportional to t^d, similar to first passage percolation on e.g. a lattice. Joint work with John Lapinskas, Johannes Lengler and Ulysse Schaller.

Tamara Kucherenko (The City University of New York, USA)
Flexibility of the Pressure Function
Abstract: We discuss the flexibility of the pressure function of a continuous potential (observable) with respect to a parameter regarded as the inverse temperature. The points of non-differentiability of this function are of particular interest in statistical physics, since they correspond to phase transitions. It is well known that the pressure function is convex, Lipschitz, and has an asymptote at infinity. We show that in a setting of one-dimensional compact symbolic systems these are the only restrictions. We present a method to explicitly construct a continuous potential whose pressure function coincides with any prescribed convex Lipschitz asymptotically linear function starting at a given positive value of the parameter. This is based on joint work with Anthony Quas.

Francois Ledrappier (CNRS, Sorbonne Université, FR)
Exact dimension of the Furstenberg measure
Abstract: We consider random walks on groups of real matrices and the stationary measures on spaces of flags. Under some conditions, we can show that these stationary measures are exact dimensional and give a formula for the dimension. This is a joint work with Pablo Lessa.

Pertti Mattila (Univ. Helsinki, FI)
Hausdorff dimension of level sets and intersections
Abstract: Let $P_\lambda : \mathbb{R}^n \to \mathbb{R}^m, \lambda \in \Lambda$, be a family of projections. If $A \subset \mathbb{R}^n$ is a Borel set with Hausdorff dimension $\dim A > m$, when is $\dim(A \cap P_\lambda^{-1}\{x\}) = \dim A - m$ for many $x \in \mathbb{R}^m, \lambda \in \Lambda$? I shall discuss some partial results and applications to the general intersections $A \cap (g(B) + z)), z \in \mathbb{R}^n, g \in O(n), A, B \subset \mathbb{R}^n$.

Radu Miculescu (Transilvania University of Brasov, RO)
The structure of fuzzy fractals generated by an orbital fuzzy iterated function system
Abstract: We will present a structure result concerning fuzzy fractals generated by an orbital fuzzy iterated function system $((X, d), (f_i)_{i\in I}, (\rho_i)_{i\in I})$. The result involves the following two main ingredients: a) the fuzzy fractal associated to the canonical iterated fuzzy function system $((I^{\mathbb{N}}; d_\Lambda), (\tau_i)_{i\in I}, (\rho_i)_{i\in I})$, where d_Λ is Baire's metric on the code space $I^{\mathbb{N}}$ and $\tau_i \colon I^{\mathbb{N}} \to I^{\mathbb{N}}$ is given by $\tau_i(\omega_1, \omega_2, \ldots) := (i, \omega_1, \omega_2, \ldots)$ for every $(\omega_1, \omega_2, \ldots) \in I^{\mathbb{N}}$ and every $i \in I$, b) the canonical projections of certain iterated function systems associated to the fuzzy fractal under consideration.

References

[1] C. Cabrelli, U. Molter, Density of fuzzy attractors: a step towards the solution of the inverse problem for fractals and other sets, Probabilistic and stochastic methods in analysis, with applications (Il Ciocco, 1991), 163-173. NATO Adv. Sci. Inst. Ser. C: Math. Phys. Sci., **372**, Kluwer Acad. Publ., Dordrecht. 1992.

[2] C. Cabrelli, B. Forte, U. Molter, E. Vrscay, Iterated fuzzy systems: a new approach to the inverse problem for fractals and other sets, J. Math. Anal. Appl., **171** (1992), 79-100.

[3] R. Miculescu, A. Mihail, I. Savu, A characterization of the fuzzy frac- tals generated by an orbital fuzzy iterated function system, it will appear in Carpathian J. Math., **38** (2022), arXiv:2203.11895v1.

[4] E. Oliveira, F. Strobin, Fuzzy attractors appearing from GIFZS, Fuzzy Sets and Systems, **331** (2018), 131-156.

William O'Regan (University of Warwick, UK)
Efficiently covering carpets with tubes
Abstract: We say a subset of Euclidean space is tube-null if for every epsilon > 0, there exists a covering of our set by tubes such that the total width of our tubes is less than epsilon. We give an example of a class of carpets involving rotations which satisfy this property.

Mark Pollicott (University of Warwick, UK)
Estimating dimension, exponents and drift
Abstract: Given a conformal iterated function scheme of contractions satisfying the open set condition the dimension of the limit set can be written implicitly using the Bowen-Ruelle formula (generalizing the classic Moran formula for affine contractions). We will discuss some recent approaches to extracting estimates on the dimension and consider some simple examples. We will also discuss related estimates for Lyapunov exponents for expanding maps and random matrix products, and also to the drift for random walks on Fuchsian groups.

Feliks Przytycki (IMPAN, PL)
No hyperbolic subsets in solenoidal sets for quadratic polynomials
Abstract: I shall discuss iteration of infinitely renormalizable quadratic polynomials of one complex variable. I will sketch a proof that there are no invariant hyperbolic subsets in J which is the intersection of orbits of small Julia sets of consecutive renormalizations. Under some additional assumptions we prove there are no invariant probability measures supported in J with positive Lyapunov exponent so the Lyapunov exponent at the critical value must be zero. This is a joint work with Genadi Levin.

Peter Raith (University of Vienna, AT)
On fractal dimensions of invariant sets for low dimensional dynamical systems
Abstract: For expanding piecewise monotonic maps $T : [0,1] \to [0,1]$ the Hausdorff dimension of invariant subsets is investigated. Here piecewise monotonic map means that there exists a finite partition $\mathcal{Z}$ into pairwise disjoint open intervals with $\bigcup_{Z\in\mathcal{Z}} \overline{Z} = [0,1]$ such that $T\big|_Z$ is continuous and strictly monotonic for all $Z \in \mathcal{Z}$. Moreover assume that for all $Z \in \mathbb{Z}$ the map $T\big|_Z$ is differentiable and its derivative can be extended to a continuous function on the closure of Z. Given a finite union U of open intervals define $A := [0,1] \setminus \bigcup_{j=0}^{\infty} T^{-j}U$. Then the Hausdorff dimension of A equals the zero of $t \mapsto p(A, T, -t \log |T'|)$, where $p(.,.,.)$ denotes the topological pressure. Results on multifractal Hausdorff dimensions of A are also presented.

Next assume that $F : [0,1]^2 \to [0,1]^2$ is a map of the form $F(x,y) := \big(Tx, g(x,y)\big)$, where $Tx := \frac{\alpha}{2} - \alpha\left|x - \frac{1}{2}\right|$ for some $\alpha \in \left[\sqrt{2}, 2\right]$. Furthermore assume that $g(x,y) := \varphi(x) + \lambda\left(y - \frac{1}{2}\right)$ for some $\lambda \in \left(0, \frac{1}{\alpha^2}\right)$ and a linear map $\varphi : [0,1] \to [\frac{\lambda}{2}, 1 - \frac{\lambda}{2}]$. Define $\Lambda := \bigcap_{n=0}^{\infty} F^n\left([0,1]^2\right)$, which can be considered as the attractor of F. A result on the Hausdorff dimension of this attractor is presented.

Ariel Rapaport (Techion, Israel Institute of Technology, IS)
Dimension of self-affine measures in $\mathbb{R}^d$
Abstract: I will present new results regarding the dimension of self-affine measures in $\mathbb{R}^d$. These rely on a new entropy increase statement for projections of self-affine measures, which is valid under standard irreducibility and proximality assumptions.

Joel Ratsaby (Ariel University, IS)
Bounded-complexity approximation of filled-Julia sets
Abstract: Consider orbits $\mathcal{O}(z,\kappa)$ of the usual fractal iterator $f_\kappa(z) := z^2 + \kappa$, $\kappa \in \mathbb{C}$, that start at an initial point $z \in K_\kappa^{(m)} \subset \hat{\mathbb{C}}$, where $\hat{\mathbb{C}}$ is the set of all rational complex numbers (their real and imaginary parts are rationals) and $K_\kappa^{(m)}$ consists of all such z whose complexity does not exceed some complexity parameter value m (the complexity of z is defined as the number of bits that suffice to describe the real and imaginary parts of z in lowest form). The set $K_\kappa^{(m)}$ is a bounded-complexity approximation of the filled-Julia set K_κ. We present a new perspective on fractals based on an analogy with Chaitin's algorithmic information theory, where a rational complex number z is the analogue of a program p, an iterator f_κ is analogous to a universal Turing machine U which executes program p, and an *unbounded* orbit $\mathcal{O}(z,\kappa)$ is analogous to an execution of a program p on U that *halts*. We define a real number Υ_κ which resembles Chaitin's Ω number, where, instead of being based on all programs p whose execution on U halts, it is based on all rational complex numbers z whose orbits under f_κ are unbounded.

We show that if Υ_κ was known, then the set $K_\kappa^{(m)}$ which approximates the filled-Julia set K_κ can be computed exactly in finite time, for any arbitrary finite complexity value m, without any heuristic or assumption (in contrast to standard approximation algorithms that use an escape-time heuristic). Hence, similar to Chaitin's Ω number, Υ_κ acts as a theoretical limit, or a 'fractal oracle number', that provides

an arbitrarily accurate complexity-based approximation of the filled-Julia set K_κ. Unlike escape-time approximations of K_κ, this complexity-based approximation is consistent, in the sense that, if $z \in K_\kappa^{(m)}$ then $z \in K_\kappa$. We provide some preliminary estimates on the necessary and sufficient value of the complexity parameter m for zooming into any arbitrarily small square region of $\hat{K}_\rho^{(m)}$.

Alex Rutar (University of St Andrews, UK)
Classifying Dimension Spectra
Abstract: Dimension spectra are continuously-parametrized families of dimensions. The two most well-studied notions are the Assouad spectrum, which interpolates between the box and Assouad dimensions, and the intermediate dimensions, which interpolate between the Hausdorff and box dimensions. These notions of dimensions are useful for understanding the geometry of inhomogeneous sets where the Hausdorff and Assouad dimensions differ, such as for various overlapping self-similar and self-affine sets. In this talk, I will first provide a brief introduction to these notions of dimension. Then I will discuss new results which give full classifications of the possible functions which are the Assouad spectra or intermediate dimensions of a subset of $\mathbb{R}^d$. These results also allow the construction of some exotic examples. The results on intermediate dimensions are joint with Amlan Banaji.

Pablo Shmerkin (University of British Columbia, CAN)
Slices and tubes through self-similar sets
Abstract: Many problems in geometric measure theory and analysis are concerned with understanding the distribution of fractal sets and measures near lower-dimensional subspaces. I will survey some (older and newer) results on the intersections of self-similar and related objects with linear slices and tubes. Part of the talk is based on a joint work with A. Pyörälä, V. Suomala and M. Wu.

Vijay Siwach (Indian Institute of Technology (IIT), Madras, IN)
Rational Spline Zipper α-Fractal Functions
Abstract: This paper introduces a novel class of C^1-rational cubic spline zipper α-fractal functions with variable scalings. For a given Hermite data set, first we construct a new class of rational cubic splines having a cubic numerator and quadratic denominator with two shape parameters, using a binary vector named signature. Then using these rational cubic splines, a class of base functions, and the theory of the zipper iterated function system, we construct C^1-rational cubic spline zipper α-fractal functions. It is shown that the proposed interpolants converge uniformly to a C^2-data generating function. We derive some sufficient conditions on the parameters to construct the positivity and monotonicity preserving interpolants. Range restriction properties of the proposed interpolants are also studied. Some graphs of proposed interpolants are plotted with the desired property.

Boris Solomyak (Bar-Ilan University, IS)
(Almost) thirty years of transversality method
Abstract: In this talk I will survey the development of the transversality method in fractal geometry, in which Karoly Simon played a crucial role, from a personal vantage point. In particular, I will touch on (i) historical development of the transversality method; (ii) applications to self-similar, self-affine, and non-linear IFS; (iii) methods for checking transversality.

Anna Soós (Babes-Bolyai University, RO)
TBA
Abstract: TBA

Adam Śpiewak (Bar-Ilan University, IS)
Typical absolute continuity for classes of dynamically defined measures
Abstract: Consider a one-parameter family of iterated function systems on the interval and a family of measures on the corresponding symbolic space, depending on the same parameter. We show that classical transversality results on the typical absolute continuity extend to this setting. Among applications are invariant measures for systems with place dependent probabilities and equilibrium measures for hyperbolic IFSs. This is joint work with Balázs Bárány, Károly Simon and Boris Solomyak.

Mohit Tripathi (Indian Institute of Technology (IIT), Madras, IN)
Fractal Interpolation for Data Set with Stable Noise
Abstract: Fractal interpolation is a modern and advanced technique for modeling scientific data. The majority of researchers in the field of fractal interpolation have focused on deterministic data sets. However, various real-world data sets contain random noise. Therefore, we also need to pay attention to fractal interpolation for noisy data sets. In this talk, we consider a data set in $\mathbb{R}^2$ that contains stable distributed noise (a generalization of Gaussian noise) on its ordinate. We briefly describe the construction of a linear recurrent fractal interpolation function with variable scaling parameters for this noisy data set. Since the interpolation data contains random noise, the interpolated values of the fractal function will also have noise. Hence, we determine the distribution of noise at interpolated values. Finally, a simulation study is presented to visualize uncertainty over the interpolated values.

Mariusz Urbański (University of North Texas, USA)
The exact value of Hausdorff dimension of escaping sets of class $\mathcal{B}$ of meromorphic functions
Abstract: We consider the subclass of class $\mathcal{B}$ consisting of meromorphic functions $f : \mathbb{C} \to \mathbb{C}$ for which infinity is not an asymptotic value and whose all poles have orders uniformly bounded from above. This class was introduced in [BwKo2012] and the Hausdorff dimension $HD(I(f))$ of the set $I(f)$ of all points escaping to infinity under forward iteration of f was estimated therein. In this paper we provide a closed formula for the exact value of $HD(I(f))$ identifying it with the critical exponent of the natural series introduced in [BwKo2012]. This exponent is very easy to calculate for many concrete functions. In particular, we construct a function from this class which is of infinite order and for which $HD(I(f)) = 0$.

Benjamin Ward (University of York, UK)
On the measure of Badly approximable sets.
Abstract: In classical Diophantine approximation the set of Badly approximable points are those whose rate of approximation, in relation to Dirichlet's Theorem, cannot be improved by an arbitrary small constant. It is a well known result that the lebesgue measure of Badly approximable points is null. The notion of Badly approximable points has natural generalizations in other n-dimensional metric spaces. In joint research with Victor Berensevich we prove that in any product space composed of a finite number of bounded separable metric spaces, each equipped with a σ-finite doubling Borel regular measure, the product measure of Badly approximable points is null. The proof of this theorem uses standard results from geometric measure theory, including a generalised Lebesgue Density Theorem and Fubini's Theorem.

Christian Wolf (The City College of New York, USA)
Computability in Dynamical Systems
Abstract: In this talk we give an overview about recent progress in the area of computability in dynamical systems. In particular, we cover results concerning the computability of the entropy and pressure, the computability of dynamically relevant sets including Julia sets and rotation sets, and the computability of certain natural invariant measures.

Caroline Wormell (CNRS, Sorbonne Université, FR)
Linear response for high-dimensional systems and mixing of Cantor sets
Abstract: The long-time statistical response of a chaotic system to dynamical perturbations is an important problem in physical sciences: estimation of the first-order ("linear") response is commonly used to reduce computational requirements when studying systems such as global climate models. Linear response is known not to exist in low-dimensional systems such as the logistic map, but the fundamental question of the existence of a linear response in non-hyperbolic high-dimensional systems, while seen in practice, remains a mathematical mystery.

When the dimension of the attractor is large, an improvement in the differentiability of the response has been conjectured by Ruelle. To probe this, we study piecewise hyperbolic maps, which are mathematically tractable models of non-hyperbolic dynamics. We prove that linear response for these maps reduces to the following property: that conditional measures of the SRB measure on singularity sets, when pushed forward by the map, converge exponentially fast to the full SRB measure.

Perhaps surprisingly, this property appears to hold for a wide range of maps. We show this numerically for the Lozi map, and use recent Fourier dimension results to prove it for certain classes of baker's

maps. Better understanding of this problem in fractal geometry will help elucidate the linear response question, justifying common practice in the physical sciences.

Alexia Yavicoli (University of British Columbia, CAN)

Thickness and a Gap Lemma in $\mathbb{R}^d$

Abstract: I will give a definition of thickness in $\mathbb{R}^d$ and present a Gap Lemma that is useful even in the totally disconnected context. Several examples and applications to patterns in fractals will be discussed.

Xintian Zhang (University of Bristol, UK)

Hausdorff dimension of eventually always hitting set of self similar IFS with SSC

Abstract: We consider the eventually always hitting set of self-similar set where the target is symbolically defined. We give a classification of the size, namely measure and Hausdorff dimension, of this eventually always hitting set via the symbolically shrinking speed of the target. This context generalizes the result of Bugeaud and Liao's result where the problem is considered on real line and specific targets. One application of the result shows that for almost all symbolic targets in sense of Bernoulli measure, we have that the Hausdorff dimension of the eventually always hitting set studied in this context is identically equally to the minimal of its topological dimension and the unique zero of a modified pressure function. Some Hausdorff dimension results of the intersection of certain eventually always hitting set and shrinking target set are also involved in this context.

3

Registered Participants

- Vishal Agrawal, *Indian Institute of Technology (BHU)*
- Pieter Allaart, *University of North Texas*
- Demi Allen, *University of Exeter*
- Simon Baker, *University of Birmingham*
- Amlan Banaji, *University of St Andrews*
- Krzysztof Barański, *University of Warsaw*
- Zoltán Buczolich, *Eötvös Loránd University*
- Alec Chamberlain Cann, *University of Bristol*
- Klaudiusz Czudek, *Nicolaus Copernicus University*
- Kenneth Falconer, *University of St. Andrews*
- Ábel Farkas, *Alfred Rényi Institute of Mathematics*
- Sandra Hayes, *The City University of New York Graduate Center*
- Yubin He, *Université Paris-Est Créteil*
- Mike Hochman, *Hebrew University of Jerusalem*
- Daniel Ingebretson, *Ben-Gurion University of the Negev*
- Taylor Jones, *University of North Texas*
- Thomas Jordan, *University of Bristol*
- Natalia Jurga, *University of St Andrews*
- Antti Käenmäki, *University of Oulu*
- Kiko Kawamura, *University of North Texas*
- Tamas Keleti, *Eötvös Loránd University*
- Gabriella Keszthelyi, *Budapest University of Technology and Economics*
- Samuel Kittle, *University of Cambridge*
- Tamás Kói, *Budapest University of Technology and Economics*
- Júlia Komjáthy, *Delft University of Technology*
- Alexey I. Korepanov, *LPSM, Paris*
- Tamara Kucherenko, *The City University of New York*
- Francois Ledrappier, *CNRS, Sorbonne Université*
- András Madai, *Silk Road Association*
- Pertti Mattila, *University of Helsinki*
- Radu Miculescu, *Transilvania University of Brasov*
- William O'Regan, *University of Warwick*
- Vilma Orgoványi, *Budapest University of Technology and Economics*
- Mark Pollicott, *University of Warwick*

- Feliks Przytyczki, *IMPAN*
- Peter Raith, *University of Vienna*
- Ariel Rapaport, *Techion, Israel Institute of Technology*
- Balázs Ráth, *Budapest University of Technology and Economics*
- Joel Ratsaby, *Ariel University*
- Tom Rush, *University of Bristol*
- Alex Rutar, *University of St Andrews*
- Csaba Sándor, *Budapest University of Technology and Economics*
- Benedict Sewell, *Alfred Rényi Institute of Mathematics*
- Pablo Shmerkin, *University of British Columbia*
- Károly Simon, *Budapest University of Technology and Economics*
- Vijay Siwach, *Indian Institute of Technology (IIT), Madras*
- Boris Solomyak, *Bar-Ilan University*
- Anna Soós, *Babes-Bolyai University*
- Adam Śpiewak, *Bar-Ilan University*
- Péter Szilvássy, *Silk Road Association*
- Mohit Tripathi, *Indian Institute of Technology (IIT), Madras*
- Sascha Troscheit, *University of Vienna*
- Mariusz Urbański, *University of North Texas*
- Imre Varga, *Budapest University of Technology and Economics*
- Benjamin Ward, *University of York*
- Christian Wolf, *The City College of New York*
- Caroline Wormell, *CNRS, Sorbonne Université*
- Tianhong Yang, *University of Bristol*
- Alexia Yavicoli, *University of British Columbia*
- Qian Zhang, *Université Paris-Est Créteil*
- Xintian Zhang, *University of Bristol*